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Diagnostic Analysis of Water Circulations in Lake Biwa*

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Abstract: A diagnostic model is developed for the study of steady water-circulations during summer in Lake Biwa. The most characteristic feature of the present model is to include the vertical friction terms in the basic equations, so that it is not necessary to assume a level of no-motion. Under no-wind condition, the velocity field is calculated from the density field obtained by successive observations of water temperature using a bathythermograph.

The comparisons of the present calculation with a dynamical calculation and direct current measurements indicate that the present model surpasses a dynamical calculation in the respects that 1) vertical circulations can be estimated and 2) the flow pattern in the deep layers can be obtained more reliably. One of important results of the present calculation is that a large vertical circulation has been found accompanied by the large counterclockwise gyre in the north basin. Preliminary results of direct current measurements by cross-boad drogues also seem to suggest the existence of the circulation.

1. Introduction

In recent years, a number of direct current measurements have been carried out both oceans and lakes. Because of technical difficulties, however, they have been fragmentary in regard to time and space. On the other hand, it is rather easy to observe the temperature and salinity fields and to estimate the density field, which is dynamically related to the velocity field. Therefore, at the present stage, it must be quite useful to extract necessary informations on the velocity field from the observed density field. A dynamical calculation is one of the method in this category and has been popularly used. It requires, however, a doubtful assumption of a level of no-motion. It is quite questionable whether this method is applicable to the case of shallow coastal sea or a closed basin such as Lake Biwa. Then, we introduce a new diagnostic model in place of a traditional dynamical calculation for estimating patterns and magnitudes of steady water-circulations in Lake Biwa.

According to observational studies on water movements in Lake Biwa (OKAMOTO and MORIKAWA, 1961a; OKAMOTO, 1968), it has been found that horizontal circulation in the surface layer of the north basin is almost stable during summer and that the currents are approximately on a geostrophic balance. On the basis of these studies, many temperature observations have been carried out and the informations of the velocity field calculated by a dynamical calculation have been increased (OKAMOTO and MORIKAWA, 1961b; KUNISHI *et al.*, 1967).

However, several fundamental and important questions have not been answered yet with regard to the estimation of the steady velocity field by the observed density field in Lake Biwa. The first question is: "Can we apply a dynamical calculation to Lake Biwa?" The second question is: "Do the observed data really represent the density field in a steady state?"

With respect to the first question, the validity of the geostrophic approximation should be examined by making observations of the current and the water density simultaneously. In the case of Lake Biwa, however, only a few trials have been made up to the present (*e.g.* OKAMOTO, 1968). Even if the circular currents in Lake Biwa could be assumed to be quasi-geostrophic, we should solve two problems to calculate the velocity field by means of a dynamical

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calculation. One of them is that the dynamic height cannot be determined in the area shallower than the depth of a level of no-motion. Another one is that a level of no-motion cannot be chosen arbitrarily in the closed basin where the total water mass should be conserved. With regard to the latter one, KUNISHI and SATO (1970) have proposed an idea to determine a level of no-motion so that the net volume transport across any vertical cross sections should be zero. However, this condition should always be fulfilled independent of a level of no-motion for steady circulations without source and sink. In any case, if a level of no-motion can be determined for convenience sake, a dynamical calculation might give only the current pattern roughly in the surface layer and no reliable informations about water movements in the layer deeper than a level of no-motion. Hence, we try to develop a new diagnostic method in which a level of no-motion is not used.

HOLLAND and HIRSCHMAN (1972) calculated circulations in the North Atlantic Ocean by means of a diagnostic model using observed density data. Since all terms are included in the horizontal momentum equations, their model requires considerably complicated and expensive calculations. Then, as the first step, we include only the terms of the vertical friction forces in the geostrophic equations because we wish to obtain more realistic solution for water-circulations especially in the deep layer. Though the details will be described in section 3, the problems mentioned above can be solved by this equation system. SARKISYAN and PASTUKHOV (1970) have applied the similar equation system to the deep ocean in which they impose the condition that flows in the middle layer are on a *pure* geostrophic balance. In our model, however, vertical friction forces are assumed to act in the whole layer because the mean value of the water depth is not quite large compared to the thickness of viscous boundary layer in Lake Biwa.

As to the second question on the steadiness, it should be noticed that actual flows in Lake Biwa consist of several dominant phenomena with various time-scale. For example, the longest period of the surface seiche is about four hours (IMASATO, 1972) and that of the internal seiche is about two days during summer (KANARI, 1973). Especially the internal seiche is on a quasi-geostrophic balance because its time-scale is longer than a inertial period (about 21 hours at Lake Biwa), and it causes a large amplitude of fluctuation to the velocity and temperature fields. In order to obtain the steady part of the density field, therefore, one must carry out continuous observations of water temperature field for a period long enough to cancel the effect of the slowest internal seiche. Such observations were carried out by the author *et al.* during summer in 1973. Using these data, patterns and magnitudes of steady water-circulations in Lake Biwa have been diagnostically estimated.

2. Observations of temperature fields

For the purpose of obtaining the steady part

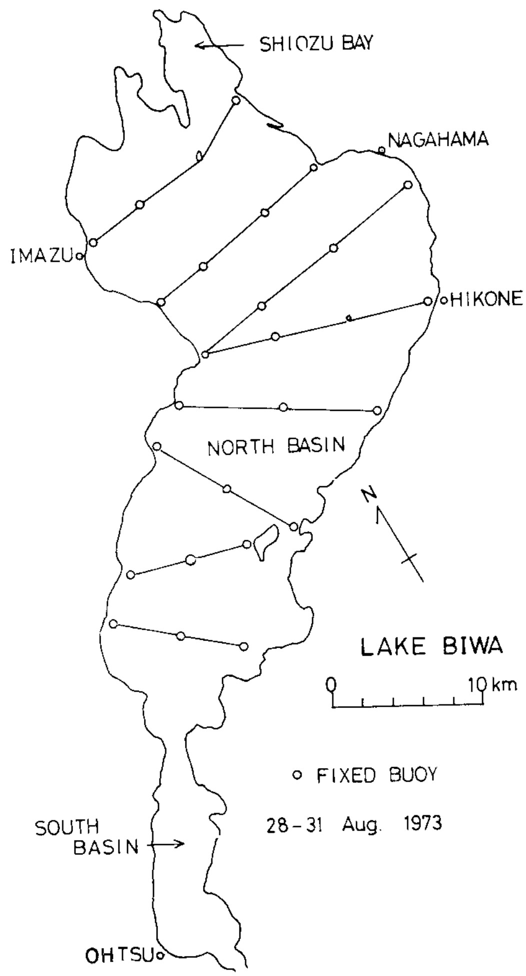

Fig. 1. Locations of observed transverse sections.

of the water density field, successive temperature measurements by a bathythermograph were made for four days between 28–31 August, 1973, each covering the almost entire region of the north basin. We set eight transverse sections in the north basin and about eight stations on each section at intervals of 1∼2 km. The locations of the transverse sections are shown in Fig. 1, where the mark ○ indicates the marker buoy station. It takes only about eight hours (one-sixth of the period of the slowest internal seiche) for each survey which consists of very dense observations over 60 stations.

Fig. 2 shows the distributions of water temperature for four days at a depth of 10 m corresponding to the top of the thermocline. In this figure dots indicate observed stations and solid curves represent isotherms. From this figure, it is quite clear that there exists a cold water region in the middle of the basin throughout four days. This region has been well-known to exist constantly during summer. Outside of this region, however, the temperature field largely fluctuates day by day; the main direction of overall temperature gradients seems to rotate counterclockwise. This temperature fluctuation might mainly be caused by the internal Kelvin wave pointed out by KANARI (1973).

Since four days of observation period is about two times longer than the period of the internal Kelvin wave, an arithmetical average of the four temperature fields might represent almost the

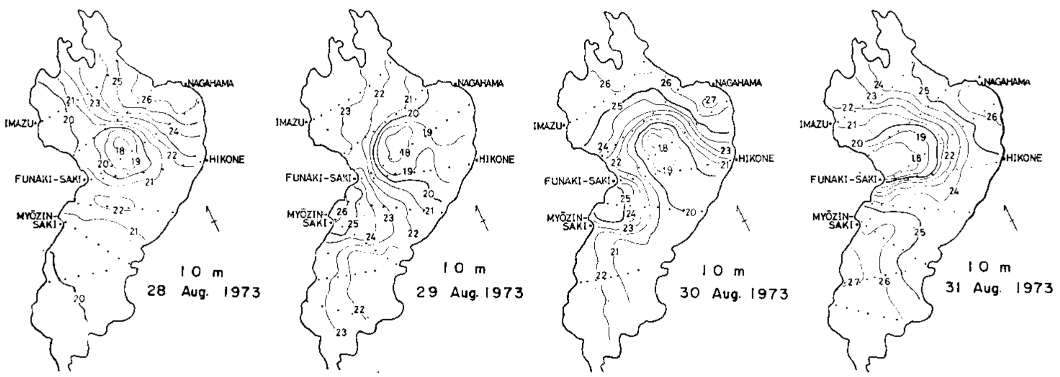

Fig. 2. Horizontal distributions of the water temperature (°C) at a depth of 10 m from 28th to 31st August 1973.

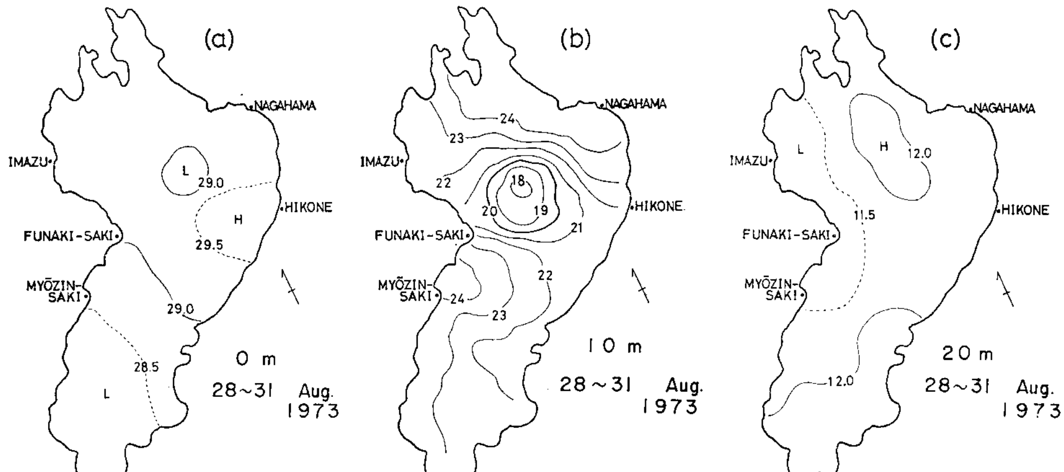

Fig. 3. Horizontal distributions of the mean water temperature (°C) at three depths; (a) 0 m, (b) 10 m and (c) 20 m.

steady part of the temperature field. Figs. 3a, b and c show the temperature distributions averaged over four days at depths of 0, 10 and 20 m, respectively. It is to be noted that the temperature field given in Fig. 3b is much smoother than that of Fig. 2. This seems to indicate that the effects of the internal seiche have been fairly removed by making averages. In the layers deeper than 20 m, the temperature shows little horizontal fluctuations and the mean values at 30, 50 and 70 m are 9.1, 7.5 and 7.1°C, respectively.

3. The method of calculation

The basic assumptions of the present model are as follows: (1) the Boussinesq and hydrostatic approximations are valid; (2) the flow is steady; (3) nonlinear terms and horizontal friction forces are neglected; (4) the lake surface is not exposed to the wind. The assumption (4) is justified by the fact that there were no dominant wind during the temperature surveys. With these assumptions, the momentum equations and the continuity equation in the Cartesian coordinate system are written as

$$f\boldsymbol{k}\times\boldsymbol{V}=-\frac{1}{\rho_0}\nabla P+\nu\frac{\partial^2\boldsymbol{V}}{\partial z^2} \qquad (1)$$

$$\frac{\partial P}{\partial z}=-\rho g \qquad (2)$$

and

$$\nabla\cdot\boldsymbol{V}+\frac{\partial w}{\partial z}=0 \qquad (3)$$

where $\nabla=\frac{\partial}{\partial x}\boldsymbol{i}+\frac{\partial}{\partial y}\boldsymbol{j}$. Here (x, y, z) form the right-handed coordinate system (z is positive upward with the origin at the lake surface), $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ are the corresponding unit vectors, f the Coriolis' parameter which can be assumed to be constant, $\boldsymbol{V}=(u, v)$ the horizontal velocity, w the vertical velocity component, P the pressure, ν the vertical eddy viscosity, g the acceleration of gravity, ρ_0 the mean density, and $\rho=\rho(x, y, z)$ the density at the point (x, y, z).

The boundary conditions, that is, no wind stress at the lake surface and no flows at the lake bottom, are given as

$$\left.\begin{aligned}&\frac{\partial\boldsymbol{V}}{\partial z}=0,\ w=0 \quad \text{at} \quad z=0\\ &\boldsymbol{V}=0,\quad w=0 \quad \text{at} \quad z=-h\end{aligned}\right\} \qquad (4)$$

where $h=h(x, y)$ is the water depth.

The pressure P is the sum of the surface pressure $P_S(x, y)$ and the internal pressure $P_I(x, y, z)$, which is given from Eq. (2) as

$$P_I(x, y, z)=-g\int_z^0\rho(x, y, z)\,dz \qquad (5)$$

Integrating Eq. (3) over depth under the boundary condition (4), we obtain

$$\nabla\cdot\int_{-h}^0\boldsymbol{V}dz=0 \qquad (6)$$

From this, the volume transport $\int_{-h}^0\boldsymbol{V}dz$ can be expressed using the stream function $\psi(x, y)$ as

$$\int_{-h}^0\boldsymbol{V}dz=-\boldsymbol{k}\times\nabla\psi \qquad (7)$$

Now let us introduce $\boldsymbol{V}_0$, $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ defined as

$$\left.\begin{aligned}f\boldsymbol{k}\times\boldsymbol{V}_0&=-\frac{1}{\rho_0}\nabla P_I+\nu\frac{\partial^2\boldsymbol{V}_0}{\partial z^2}\\ f\boldsymbol{k}\times\boldsymbol{V}_1&=\quad\frac{1}{\rho_0}\boldsymbol{i}\quad+\nu\frac{\partial^2\boldsymbol{V}_1}{\partial z^2}\\ f\boldsymbol{k}\times\boldsymbol{V}_2&=\quad\frac{1}{\rho_0}\boldsymbol{j}\quad+\nu\frac{\partial^2\boldsymbol{V}_2}{\partial z^2}\end{aligned}\right\} \qquad (8)$$

The velocities $\boldsymbol{V}_0$, $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ can be readily solved using the boundary condition (4), then, $\boldsymbol{V}$ is written in the form:

$$\boldsymbol{V}=\boldsymbol{V}_0-\boldsymbol{V}_1\frac{\partial P_S}{\partial x}-\boldsymbol{V}_2\frac{\partial P_S}{\partial y} \qquad (9)$$

Substituting Eq. (7) into the integral form of Eq. (9), we obtain the following equation.

$$-\boldsymbol{k}\times\nabla\psi=\int_{-h}^0\boldsymbol{V}_0dz -\frac{\partial P_S}{\partial x}\int_{-h}^0\boldsymbol{V}_1dz-\frac{\partial P_S}{\partial y}\int_{-h}^0\boldsymbol{V}_2dz \qquad (10)$$

Eliminating P_S by making cross differentiation of the two components of Eq. (10), a second-order partial differential equation with rerard to ψ is obtained as

$$\frac{\partial}{\partial x}\left\{F(h)\frac{\partial\psi}{\partial x}\right\}+\frac{\partial}{\partial y}\left\{F(h)\frac{\partial\psi}{\partial y}\right\} +\frac{\partial}{\partial x}\left\{G(h)\frac{\partial\psi}{\partial y}\right\}-\frac{\partial}{\partial y}\left\{G(h)\frac{\partial\psi}{\partial x}\right\} =\sigma\left(h,\ \int_{-h}^0\boldsymbol{V}_0dz\right) \qquad (11)$$

The complete forms of F, G and σ are shown in Appendix. When the stream function ψ is obtained by solving numerically Eq. (11) with the boundary condition of $\psi=0$ on the lake shores, $\int_{-h}^{0} V dz$ and ∇P_S are calculated, in turn, from Eqs. (7) and (10) succesively. Finally, the complete horizontal velocity $\boldsymbol{V}$ is calculated from Eq. (9) and the vertical velocity component w can be obtained from Eq. (3) under the boundary condition (4).

The calculated area of Lake Biwa is shown in Fig. 4, where the south basin and Shiozu Bay are excluded because they might give little effects to the large-scale circulation in the north basin. As shown in this figure, the north basin has been divided into 42×25 square meshes of 1 km intervals.

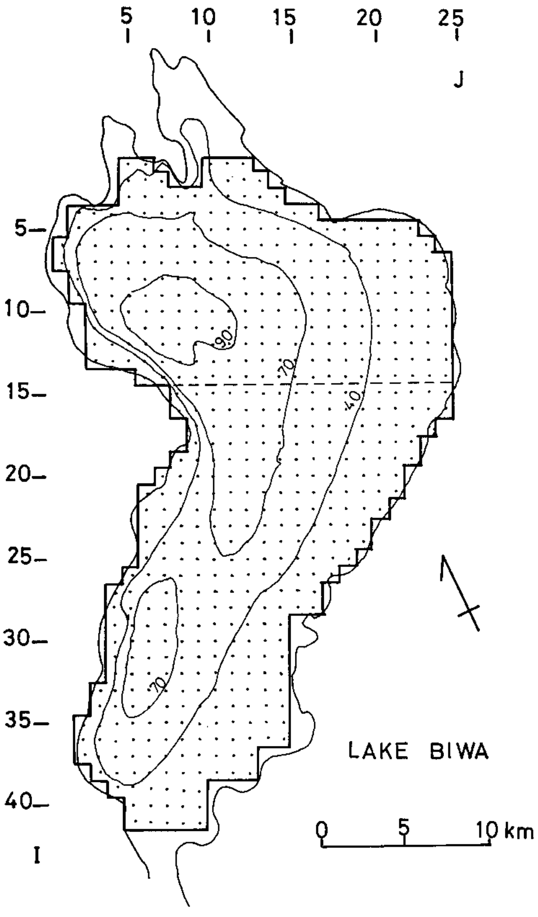

Fig. 4. Model basin of Lake Biwa with grid of 1 km square. Some topographic contours labeled in meter are also shown.

With regard to the vertical direction, the bottom Ekman layer should be resolved by fine vertical grid in order to solve Eq. (8) numerically. According to the Ekman's theory, the thickness of the viscous boundary layer D is expressed as

$$D=\pi\sqrt{2\nu/f}$$

which depends on the vertical eddy viscosity ν. TAKAHASHI (1957) has shown that the vertical eddy diffusivity K_z in Lake Biwa ranges between 0.1 and 1 $cm^2 s^{-1}$. If we assume the vertical eddy Prandtle number $\nu/K_z=10$, the viscosity ν ranges between 1 and 10 $cm^2 s^{-1}$. Then, the value of thickness D is around 10 m (f at this latitude is $8.4\times10^{-5} s^{-1}$). From these considerations, the vertical grid interval was chosen 1 m so that the bottom Ekman layer and the complicated bottom topography could be resolved sufficiently.

The density data averaged over four days were interpolated to every grid point by assuming that the effect of each observed station was inversely proportional to the distance. The basic calculation was performed with $\nu=5\ cm^2 s^{-1}$; subsidiary calculations both with $\nu=1$ and $10\ cm^2 s^{-1}$ were also made to see the effects of ν-variations. In every case, the value of ν was assumed to be constant all over the lake. Since the calculations showed that these three cases would give nearly the same circulation patterns, only the case for $\nu=5\ cm^2 s^{-1}$ will be discussed in the following section.

4. Result and discussion

Figs. 5a, b, c and d show the distributions of the calculated horizontal velocity $\boldsymbol{V}$ at the depths of 0, 10, 20 and 40 m, respectively. At the lake surface, the steady water-circulations consist of four gyres, that is, clockwise, counterclockwise, clockwise and counterclockwise in turn from the north to the south. Among them, the counterclockwise gyre **A**, situated at the cold water region in Fig. 3b, is the largest and the maximum value of the velocity is 40 $cm\ s^{-1}$. At the depth of 10 m, these four gyres are also found, though the current pattern is somewhat different from that of the lake surface; the magnitude of the flow in the gyre **A** decreases to a half or less compared with that of the lake surface, whereas the other three gyres maintain nearly the same magnitudes.

At the depth of 20 m, which corresponds to the bottom of the thermocline, the gyre **A** cannot be found any longer. Namely the gyre **A** exists only in the epilimnion (layer above the thermocline) as has been pointed out by OKAMOTO (1968) from his direct current measurements. At the depth of 40 m, the southward flows along the topographic contours visible near the western shore are dominant. Since the horizontal gradient of water density is very small at this depth, the flow may be strongly affected by the bottom topography. It has been considered that the oscillatory flow caused by internal seiches are dominant and the steady flow is very weak in the deep layer (*e.g.* OKAMOTO, 1968). The present result, however,

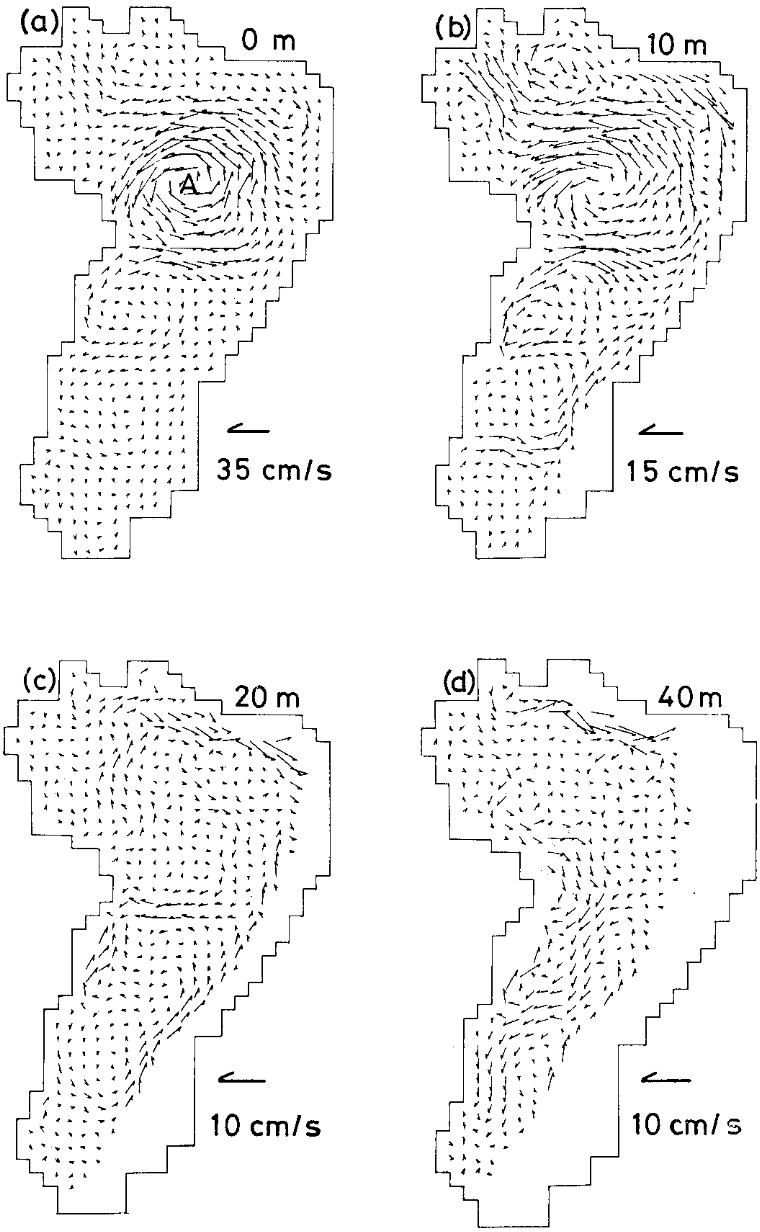

Fig. 5. Horizontal distributions of the horizontal velocity vector $\boldsymbol{V}$ at four depths; (a) 0 m, (b) 10 m, (c) 20 m and (d) 40 m.

indicates that the magnitude of the velocity in the deep layer (the maximum value is 9 cm s^{-1}) is not so small, though it has been decreased to about one-fourth of that of the surface layer.

Now let us examine the flow pattern of the gyre **A** in the vertical section along the line of I=14 (see Fig. 4), which passes the center of the gyre **A**. Fig. 6a shows the distribution of the velocity component perpendicular to this cross section. It is clear from this figure that there is an intense vertical shear of the horizontal velocity in the gyre **A**. Fig. 6b is the same as Fig. 6a but it has been obtained by means of a dynamical calculation using the same density data. A level of no-motion has been assumed to be 15 m after KUNISHI and SATO (1970), whose idea has been already mentioned above.

From the comparison between these two figures, it has been found that the flow pattern is similar in the epilimnion, but not in the hypolimnion (layer below the thermocline). With regard to the deep layer, Fig. 6a shows that the current directions are almost southward, whereas Fig. 6b shows that there is a clockwise gyre. This difference might be mainly due to the vertical friction forces and partly due to the setting of a level of no-motion to be the rather shallow depth. It is quite questionable that horizontal flows are absent anywhere at the depth of 15 m which corresponds to the thermocline and that fairly strong flows exist near the lake bottom.

On the other hand, the results obtained by the present diagnostic model might be more reliable because we considered the vertical friction forces and no doubtful concept such as a level of no-motion. Hence, a dynamical calculation may be only applicable to estimate roughly flow patterns with strong baroclinicity such as gyre **A**. From the point of view that we have little knowledge about the flow in the deep layer of Lake Biwa, the present results should be useful as one of the basic data to estimate the dispersion of dissolved or suspended matter in the lake water.

Further let us examine quantitatively the differences between the present result and those of a dynamical calculation by estimating the role of the friction forces. In order to evaluate the effects of the friction forces, we took the deviation of the calculated velocity $\boldsymbol{V}$ from the *pure* geostrophic velocity $\boldsymbol{V}_g$, defined as $\boldsymbol{V}_g = \boldsymbol{k} \times \nabla P/(\rho_0 f)$. We call this deviation $\hat{\boldsymbol{V}} = \boldsymbol{V} - \boldsymbol{V}_g$ *the velocity anomaly*. The horizontal divergence of the velocity $\boldsymbol{V}$ is not zero because of the existence of the vertical friction forces terms, whereas $\boldsymbol{V}_g$ is free from divergence. Therefore, *the velocity anomaly* $\hat{\boldsymbol{V}} = (\hat{u}, \hat{v})$ should contain the whole divergence of $\boldsymbol{V}$.

Figs. 7a, b and c show the distributions of *the velocity anomaly* $\hat{\boldsymbol{V}}$ at the depths of 0, 10 and 20 m, respectively. From these figures it is quite clear that the convergence point at the lake surface and the divergence point at the depth of 10 m correspond to the center of the gyre **A**. At the depth of 20 m, there are no remarkable features in the distributions of *the*

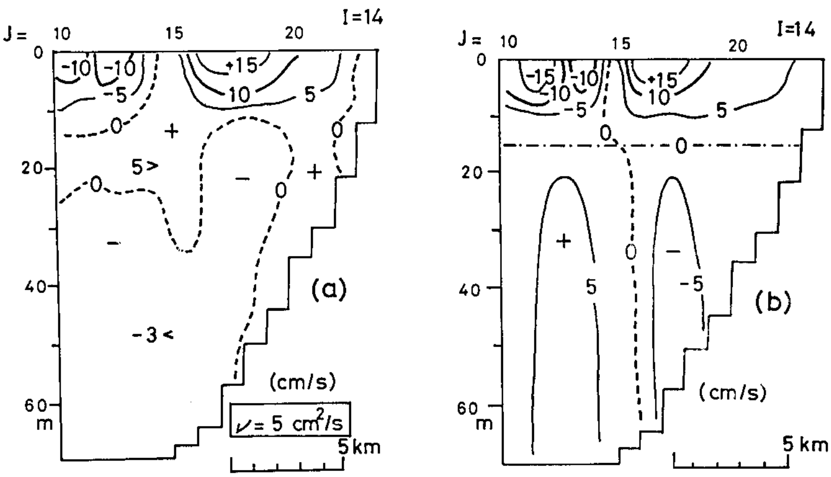

Fig. 6. Vertical distribution of the velocity perpendicular to the vertical cross section of I=14. (a) velocity calculated by the diagnostic model, (b) velocity by a dynamical calculation.

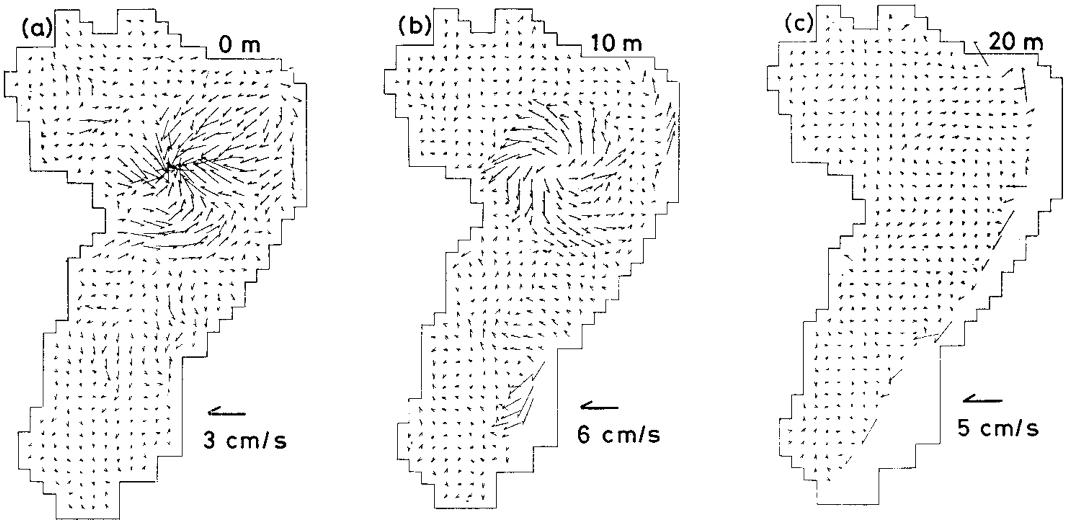

Fig. 7. Horizontal distributions of *the velocity anomaly* vectors $\hat{V}$ at three depths; (a) 0 m, (b) 10 m and (c) 20 m.

velocity anomaly.

Fig. 8 shows the distribution of $(\hat{v}, w)$ in the same vertical cross section as in Fig. 6. In this figure, we can clearly see a large vertical circulation in the epilimnion. The divergence of the flow is strongest near the depth of 15 m, which corresponds to the bottom of the gyre **A** and also the middle layer of the thermocline. The maximum value of the vertical velocity w is 3.0×10^{-2} cm s^{-1}, which is much larger than was expected.

It is interesting that the sense of the vertical circulation is the same as that obtained by OONISHI (1975). Introducing the concept of *the topographic heat accumulation effect*, he has shown that the inward heat flux across the lake surface induces a vertical circulation like this and consequently an counterclockwise horizontal circular current is formed by the Coriolis effect. In this sense, this vertical circulation should play very important roles in the formation and maintenance of the gyre **A**, as well as in the heat budget and in the dispersion of material in Lake Biwa. Therefore, we must verify its existence from observations as soon as possible. We carried out a preliminary measurement of both current and water temperature on August 29, 1975 and results are presented below.

We used the cross-boad drogues as the method of current measurements. The usefulness of this method in Lake Biwa has been referred by

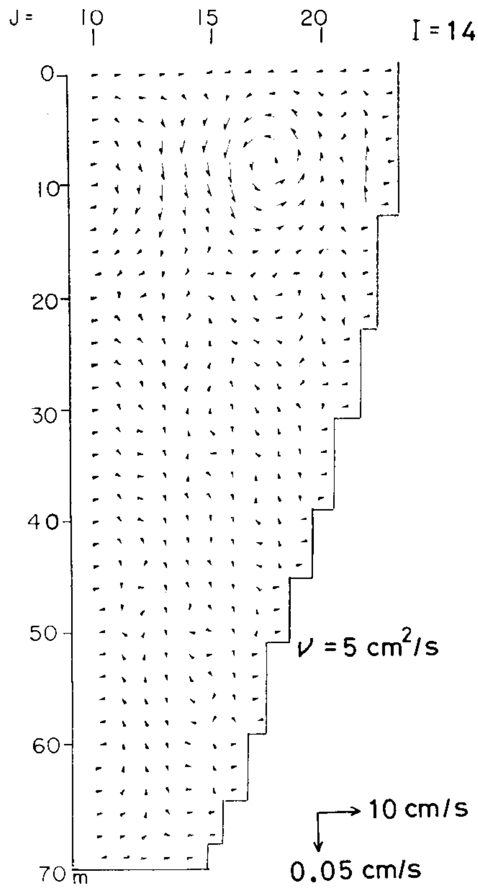

Fig. 8. The distribution of $(\hat{v}, w)$ vectors in the vertical cross section of I=14.

OKAMOTO (1964, 1968). The measurements were carried out at two stations, located at the north side (Stn. FA) and the west side (Stn. FB) of the center of the gyre **A**. The depths of the measurements are 1, 7.5, 15, 25 and 40 m. At each station, the drogues were tracked for about two hours. In addition, in order to calculate the geostrophic component of the flow by a dynamical calculation, we observed the vertical distributions of water temperatures by a bathythermograph at four points about 500 m away from the station.

Fig. 9 shows the observed velocities and the geostrophic velocities calculated from the temperature data. On making dynamical calculation, the depth of a level of no-motion was chosen at the lake bottom for convenience sake. In this figure, the directions of observed velocities in the surface layer at two stations seem to indicate the existence of a counterclockwise gyre. From the comparison of the observed velocities with the calculated geostrophic velocities, it is found that a dynamical calculation has made overestimations of velocities for the surface layer, which might be partly due to the underestimates of horizontal distances between the points of temperature observations.

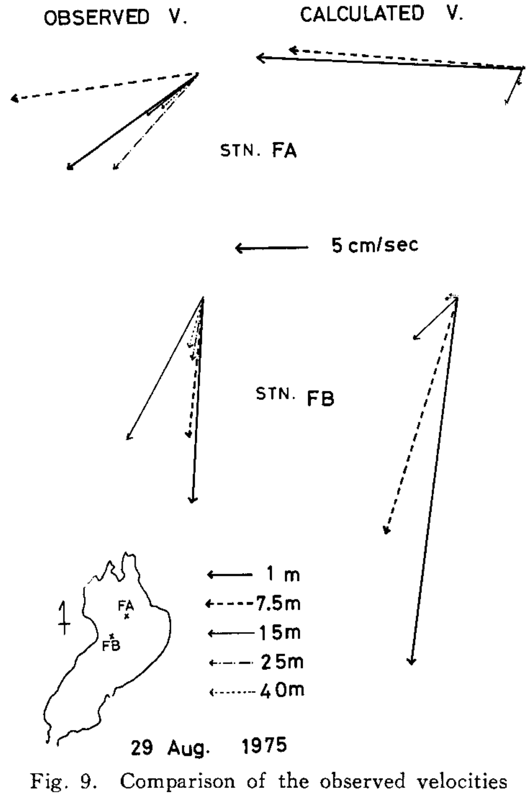

Fig. 9. Comparison of the observed velocities (the left) and the calculated geostrophic velocities (the right) at two stations.

It is very interesting that the directions of the observed velocities in the surface layer (1, 7.5 m) are deflected to the left of those of the geostrophic velocities at both stations. This means that the deviations of the observed velocities from the geostrophic, which correspond to *the velocity anomalies* defined above, direct to the center of the gyre. This is consistent at least qualitatively with the result of the present calculations (Fig. 7a) and seems to suggest the existence of the vertical circulation given in Fig. 8. We are preparing more comprehensive observations to confirm the present results.

Finally, we should like to point out that the present diagnostic model will be applicable not only to Lake Biwa but also to various fields of the shallow coastal sea for estimating the so-called density concerned currents from water temperature and salinity observations.

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Appendix

The two components of Eq. (10) can be written as

$$\left.\begin{aligned} \frac{\partial\psi}{\partial y} &= Q_{0x} - Q_{1x}\frac{\partial P_S}{\partial x} - Q_{2x}\frac{\partial P_S}{\partial y} \\ -\frac{\partial\psi}{\partial x} &= Q_{0y} - Q_{1y}\frac{\partial P_S}{\partial x} - Q_{2y}\frac{\partial P_S}{\partial y} \end{aligned}\right\} \quad \text{(A-1)}$$

where

$$\left.\begin{aligned} \boldsymbol{Q}_0 &= Q_{0x}\boldsymbol{i} + Q_{0y}\boldsymbol{j} = \int_{-h}^{0} \boldsymbol{V}_0 dz \\ \boldsymbol{Q}_1 &= Q_{1x}\boldsymbol{i} + Q_{1y}\boldsymbol{j} = \int_{-h}^{0} \boldsymbol{V}_1 dz \\ \boldsymbol{Q}_2 &= Q_{2x}\boldsymbol{i} + Q_{2y}\boldsymbol{j} = \int_{-h}^{0} \boldsymbol{V}_2 dz \end{aligned}\right\} \quad \text{(A-2)}$$

From the definitions of $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ in Eq. (8), it follows $\boldsymbol{V}_2 = \boldsymbol{k} \times \boldsymbol{V}_1$ and

$$\left.\begin{aligned} Q_{2x} &= -Q_{1y} \\ Q_{2y} &= \quad Q_{1x} \end{aligned}\right\} \quad \text{(A-3)}$$

Using these relations, Eq. (A-1) can be written as

$$\left.\begin{aligned} Q_{1x}\frac{\partial P_S}{\partial x} - Q_{1y}\frac{\partial P_S}{\partial y} &= Q_{0x} - \frac{\partial\psi}{\partial y} \\ Q_{1y}\frac{\partial P_S}{\partial x} + Q_{1x}\frac{\partial P_S}{\partial y} &= Q_{0y} + \frac{\partial\psi}{\partial x} \end{aligned}\right\} \quad \text{(A-4)}$$

Then, $\frac{\partial P_S}{\partial x}$ and $\frac{\partial P_S}{\partial y}$ can be solved from Eq. (A-4), such as

$$\begin{aligned} \frac{\partial P_S}{\partial x} &= F(h)\left(Q_{0x} - \frac{\partial\psi}{\partial y}\right) + G(h)\left(Q_{0y} + \frac{\partial\psi}{\partial x}\right) \\ \frac{\partial P_S}{\partial y} &= F(h)\left(Q_{0y} + \frac{\partial\psi}{\partial x}\right) - G(h)\left(Q_{0x} - \frac{\partial\psi}{\partial y}\right) \end{aligned} \quad \text{(A-5)}$$

where $F(h) = Q_{1x}/(Q_{1x}^2 + Q_{1y}^2)$ and $G(h) = Q_{1y}/(Q_{1x}^2 + Q_{1y}^2)$ are the functions only depended on the water depth h. By eliminating P_S by making cross differentiation of Eq. (A-5), Eq. (11) is obtained and

$$\sigma = \frac{\partial}{\partial x}\{-F(h)Q_{0y} + G(h)Q_{0x}\} + \frac{\partial}{\partial y}\{F(h)Q_{0x} + G(h)Q_{0y}\}$$

びわ湖湖流の診断モデル

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要旨: 力学計算に代る新たな方法として診断モデルを用いた数値計算によって夏季のびわ湖における定常的な湖流を調べた．このモデルの特徴は地衡流の式に鉛直粘性項を付け加えたことで，従来の力学計算における無流面の仮定を必要としない．4日間連続して行なったBTによる水温観測で得られた密度場から無風の条件下で流速場を計算した．

計算の結果を力学計算と直接測流の結果と比較した．その結果，このモデルは鉛直循環流が評価できることと，深層における流れの評価をより信頼できるものにした二点において力学計算よりすぐれていることがわかった．また，計算の結果から，北湖の大きな反時計回りの環流には大規模な鉛直循環流の伴っていることが予見され，抵抗板を用いた直接測流によってこの鉛直循環流の存在を示唆する結果が得られた．

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